AD-A273 428

‖‖‖‖‖‖‖‖‖‖‖‖‖‖‖‖‖

JMENTATION PAGE

On s est mated to average 1 nour per response including the time for reviewing instructions, searching existing data s ering and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of uding this burden to Washington Headquarters Services. Directorate for Information Operations and Reports, 1215 Jefferson and to the Office of Management and Budget. Paperwork Reduction Project (0704-0188). Washington, DC 20503

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
		FINAL/01 JUN 90 TO 31 AUG 93

4. TITLE AND SUBTITLE	5. FUNDING NUMBERS
DYNAMIC MODELING OF JOINT LIFETIMES OF MARGINALIZATION AND OF MULTI-UNIT MINIMAL REPAIRS (U)	
6. AUTHOR(S) Professor Moshe Shaked	2304/A6 AFOSR-90-0201

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
University of Arizona Dept of Mathematics Tucson AZ 85721	AFOSR-TR-

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING / MONITORING AGENCY REPORT NUMBER
AFOSR/NM 110 DUNCAN AVE, SUITE B115 BOLLING AFB DC 20332-0001	AFOSR-90-0201

DTIC ELECTE DEC 07 1993 D

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT	12b. DISTRIBUTION CODE
APPROVED FOR PUBLIC RELEASE: DISTRIBUTION IS UNLIMITED	UL

13. ABSTRACT (Maximum 200 words)

Research under this grant consisted of: (1) An introduction and a study of multivariate mean residual life functions, (2) A study of some dynamic notions of multivariate aging, (3) Modeling dynamic conjunction, separation, and marginalization of reliability systems, and (5) A further study of the multivariate notions of minimal repairs.

14. SUBJECT TERMS			15. NUMBER OF PAGES
			6
			16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR(SAME AS REPORT)

THE UNIVERSITY OF

ARIZONA

TUCSON ARIZONA

Department of Mathematics
Business Office
(602)621-1486 or 4344

Building #89
Tucson, Arizona 85721
(602)621-6893
FAX (602)621-8322

COVER SHEET FOR FACSIMILE TRANSCEIVER LOCATED IN
THE MATHEMATICS DEPARTMENT

FAX TELEPHONE NUMBER (602) 621-8322

TO: Sandra Hudson, AFOSR , Grants Office

FROM: Faye Villalobos, Business Manager , Mathematics, University of Arizona

COVER SHEET + 5

DATE: October 19, 1993

RE: Final Technical Report, AFOSR 90-0201,
Dr. Moshe Shaked, Principal Investigator

MESSAGE:

Attached is the Final Report. Again we apologize for causing your office any inconvenience

xc: Mark Bugelwicz, Sponsored Projects

Accesion For		
NTIS CRA&I	☒	
DTIC TAB	☐	
Unannounced	☐	
Justification		
By		
Distribution /		
Availability Codes		
Dist	Avail and /or Special	
A-1		

DTIC QUALITY INSPECTED 3

93-29718

‖‖‖‖‖‖‖‖‖‖‖‖‖‖‖

93 12 6 027

FINAL TECHNICAL REPORT

Grant No: AFOSR-90-0201

Title: Dynamic modeling of joint lifetimes, of marginalization and of multi-unit minimal re-
              pairs.

Due date: 30 June 1993

The proposal for Grant AFOSR-90-0201 consisted of the following parts:

(1)   An introduction and a study of multivariate mean residual life functions.

(2)   A study of some dynamic notions of multivariate aging.

(3)   Modeling dynamic conjunction, separation, and marginalization of reliability systems.

(4)   A further study of the multivariate notions of minimal repairs.

We have achieved most of the objectives of the proposal during the last three years.

(1) Consider a multi-unit system consisting of a number of components. In order to predict and
study the stochastic behaviour of the system one usually need to model the joint distribution of
the lifetimes of the components. There are several ways in which this can be done. The clas-
sical approach is to simply postulate a "reasonable" multivariate joint distribution and hope that
the postulated distribution indeed corresponds closely to the real life situation which it tries to
describe. Another common approach is to postulate the Laplace transform of the "reasonable"
life distribution. Our approach in the last few years has been to postulate the underlying life
distribution through the intuitive notion of the multivariate conditional hazard rate functions.
We have shown that the multivariate conditional hazard rate functions uniquely determine the
joint distribution of the underlying lifetimes and this justifies the use of the multivariate condi-
tional hazard rate functions for the purpose of modeling. We recently have suspected that a
similar modeling procedure can be introduced by the use of the multivariate conditional mean
residual life functions rather than the multivariate conditional hazard rate functions. It turns
out that indeed this is possible in many situations, though with some exceptions. The results
of our investigations in this area are described in [1]. A critical comparison of these two dis-
tinct, though similar, approaches can be found in [2]. We have also continued our study of
the multivariate conditional hazard rate functions. The use of the multivariate conditional haz-
ard rate functions for the purpose of simulation is described in [3]. A discrete version of these
functions is introduced and studied in [4]. The latter paper is based on a part of the Ph. D.
dissertation of the doctoral student, Jose Valdez-Torres, supervised by one of the PI's (Moshe
Shaked). In [5] we introduce and study a model of failures of units with discrete lifetimes.
We suppose that a unit has a sequence of tasks to perform and that its lifetime is measured by
the number of tasks performed before its final, fatal failure. Upon a failure the unit may be re-
paired (with some probability) and then it is given another chance to perform the current task.

-1-

The unit dies when (with some probability) a repair cannot be completed. We derive some stochastic comparisons of pairs of such models. The stochastic comparisons are then applied for obtaining results regarding the inheritance of several aging properties by the repaired unit. Various examples illustrate the applicability of the model. Some variants of the model of this paper can be viewed as discrete analogs of the notion of imperfect repair. The discrete multivariate hazard rate functions (which are closely related to the multivariate mean residual life functions) are the basic technical tool that is used in this paper. Another work in this area is done in [6]. Consider a reliability system consisting of n components. Suppose that the failures and the repair completions of the components can occur only at positive integer-valued times. At any time k each component can be in one of two states: up (i.e., working) or down (i.e., failed and in repair). The system state is also either up or down and it depends on the states of the components through a coherent structure function. In this paper we formulate mathematically the above model and we derive some of its properties. In particular, we identify conditions under which the first failure times of two such systems can be stochastically ordered. A variety of special cases is used in order to illustrate the applications of the derived properties of the model. Some instances in which the times of first failure have the NBU (=new better than used) property are pointed out. This work is based on a part of the Ph. D. dissertation of the doctoral student, Haolong Zhu, supervised by one of the PI's (Moshe Shaked).

(2) Most of the univariate notions of aging such as logconcavity, IFR, NBU etc. can be characterized by some kind of a univariate stochastic monotonicity of the mean residual lives of the corresponding underlying components. We have recently have developed several dynamic notions of multivariate stochastic orders. Having such orders it is natural to try to apply them for the purpose of developing new useful and intuitive notions of multivariate aging. This has been done in [11]. A discrete version of these multivariate stochastic orders has been developed in [8]. The latter paper is based on another part of the Ph. D. dissertation of Jose Valdez-Torres. Having these discrete multivariate stochastic notions it is now possible to introduce and study discrete analogs of the multivariate aging notions which are studied in [5]. This was done by a doctoral student, Jose Rocha, at the University of Arizona, under the supervision of one of the PI's (Moshe Shaked). Another related study about characterization of aging notions through block replacement policies is described in [9]. The latter paper is based on another part of the Ph. D. dissertation of Haolong Zhu. In [5] these notions play an important role in the quest for understanding the aging properties of the distributions of the actual lifetimes of the repaired components. In [10] we obtain stochastic comparisons of some quite general wear processes. Let $\{N_A(t), t \geq 0\}$ and $\{N_B(t), t \geq 0\}$ be two counting processes with respective independent interarrivals A_i and B_i, and let $\{X_i\}$ and $\{Y_i\}$ be two sequences of independent non-negative random variables. Consider the two compound processes which are based on these variables. Then it is well known that, under some assumptions, if the A_i are in-

dependent of the X_i, and if the B_i are independent of the Y_i, then the resulting processes can be compared stochastically. In this paper we provide conditions under which this stochastic order relation holds even under weaker conditions. In fact, we derive the stochastic order relationships among processes which are much more general than the cumulative ones described above. For example, it is not necessarily assumed that the A_i are independent of the X_i, or that the B_i are independent of the Y_i. Examples which illustrate the applications of the theory are included.

(3) Consider a set of components which usually work together as parts in some complex system. It is often of interest to study the behaviour of one of the components (or of a subset of the components) "separately" from the other components. This leads one to consider the marginal distribution of the particular component or of the set of components. However, formally, the classical definition of marginal distributions is not always meaningful in some applications in reliability theory. This difficulty with the classical definition of marginal distribution led us to study a new notion of marginal distributions. The new notion is based on the notion of multivariate imperfect repair which the PI's have developed earlier. The study is described in [7]. The study shows how to formalize different interpretations of "separability" of components. In particular, the study also shows how to model conjunctions of reliability systems (that is, how to have a formal way of modeling combinations of components from different systems at some point in time in which they are not new anymore and may have different ages).

(4) The use of the notion of multivariate minimal repairs has already been mentioned above as a tool for modeling dynamic conditional marginal distributions. Minimal repairs are also of interest in situations of tight budgets which do not permit a perfect replacement each time a component fails. In such case the component is repaired so that it is brought back to a state of being functional, but, in general, it is not as good as new then. In some such situations the budget may not even permit an unlimited number of minimal repairs. It may suffice only for some fixed number of minimal repairs which in such a case need to be allocated for the various components of the system. The question of what is the optimal allocation then arises. In [12] we have studied questions of this kind, and we have found the optimal allocations in a variety of tight budget situations. Not only minimal repairs have been considered, but also the optimal policies of assigning various kinds of standby replacements. The notions of minimal and imperfect repairs are useful as well in other recent studies done under this this Grant. For example, in [5] these notions are used as a model for repair of discrete time components. In fact, one can view the results in [5] as a possible version of imperfect repair in discrete time.

Some work that has not been described in the proposal has also been done under this Grant. In [13], [14] and [15] various notions of stochastic convexity and stochastic majorization are studied in detail. In [16] and [17] it is shown that often (but not always) the statistical information content of positively dependent random variables is larger than the statistical information content of comparable independent random variables.

REFERENCES

[1] Shaked, M. and Shanthikumar, J. G. (1991), Dynamic multivariate mean residual life functions, *Journal of Applied Probability*, 28, 613-629.

[2] Shaked, M. and Shanthikumar, J. G. (1993), Multivariate conditional hazard rate and mean residual life functions and their applications, *Reliability and Decision Making* (Eds: R. E. Barlow, C. A. Clarotti and F. Spizzichino), 137-155.

[3] Shaked, M. and Shanthikumar, J. G. (1991), Dynamic construction and simulation of random vectors, *Topics in Statistical Dependence* (Eds: H. W. Block, A. R. Sampson and T. H. Savits), 415-433.

[4] Shaked, M., Shanthikumar, J. G. and Valdez-Torres, J. B. (1993), Discrete hazard rate functions, *Computers and Operations Research*, to appear.

[5] Rocha-Martinez, J. M. and Shaked, M. (1992), A discrete-time model of failure and repair, Technical Report, Department of Mathematics, University of Arizona.

[6] Shaked, M., Shanthikumar, J. G. and Zhu, H. (1993), Discrete repairable reliability systems, *Naval Research Logistics*, 40, 769--780.

[7] Shaked, M. and Shanthikumar, J. G. (1993), Dynamic conditional marginal distributions in reliability theory, *Journal of Applied Probability*, 30, 421--428.

[8] Shaked, M., Shanthikumar, J. G. and Valdez-Torres, J. B. (1991), Discrete probabilistic orderings in reliability theory, Technical Report, Department of Mathematics, University of Arizona.

[9] Shaked, M. and Zhu, H. (1992), Some results on block replacement policies and renewal theory, *Journal of Applied Probability*, 29, 932-946.

[10] Pellerey, F. and Shaked, M. (1993), Stochastic comparison of some wear processes, *Probability in the Engineering and Informational Sciences*, 7, 421-435.

[11] Shaked, M. and Shanthikumar, J. G. (1991), Dynamic multivariate aging notions in reliability theory, *Stochastic Processes and Their Applications*, 38, 85-97.

[12] Shaked, M. and Shanthikumar, J. G. (1992). Optimal allocation of resources to nodes of parallel and series systems, *Advances in Applied Probability*, **24**, 894-914.

[13] Li, H. and Shaked, M. (1993), Stochastic majorization of stochastically monotone families of random variables, *Advances of Applied Probability*, to appear.

[14] Li, H. and Shaked, M. (1993). Stochastic convexity and concavity of Markov processes, *Mathematics of Operations Research*, to appear.

[15] Shaked, M. and Shanthikumar, J. G. (1991), Regular, sample path and strong stochastic convexity: A review, *Stochastic Orders and Decision under Risk* (Eds: K. Mösler and M. Scarsini), 320-333.

[16] Shaked, M. and Tong, Y. L. (1992), Comparison of experiments via dependence of normal variables with a common marginal distribution, *Annals of Statistics*, **20**, 614-618.

[17] Shaked, M. and Tong, Y. L. (1992), Comparison of experiments of some multivariate distributions with a common marginal, *Stochastic Inequalities* (Eds: M. Shaked and Y. L. Tong), 388-398.